

## Q3: (Tutorial) Warm Up: Recursive Multiplication

These exercises are meant to help refresh your memory of topics covered in lecture and/or lab this week before tackling more challenging problems.

Write a function that takes two numbers `m` and `n` and returns their product. Assume `m` and `n` are positive integers. Use **recursion**, not `mul` or `*`!

Hint:  $5 * 3 = 5 + (5 * 2) = 5 + 5 + (5 * 1)$ .

For the base case, what is the simplest possible input for multiply?

For the recursive case, what does calling `multiply(m - 1, n)` do? What does calling `multiply(m, n - 1)` do? Do we prefer one over the other?

**Challenge:** Try to implement the multiply function tail recursively.

## Q4: (Tutorial) Recursive Hailstone

Recall the `hailstone` function from Homework 1. First, pick a positive integer `n` as the start. If `n` is even, divide it by 2. If `n` is odd, multiply it by 3 and add 1. Repeat this process until `n` is 1. Write a recursive version of `hailstone` that prints out the values of the sequence and returns the number of steps.

```
def hailstone(n):
    """Print out the hailstone sequence
    starting at n, and return the number of elements in the
    sequence.
    >>> a = hailstone(10)
    10
    5
    16
    8
    4
    2
    1
    >>> a
    7
    """
    "*** YOUR CODE HERE ***"
```

Hint: When taking the recursive leap of faith, consider both the return value and side effect of this function.

## Q6: (Tutorial) Count K

Consider a special version of the `count_stairways` problem, where instead of taking 1 or 2 steps, we are able to take up to and including `k` steps at a time. Write a function `count_k` that figures out the number of paths for this scenario. Assume `n` and `k` are positive.

```
def count_k(n, k):
    """ Counts the number of paths up a flight of n stairs
    when taking up to and including k steps at a time.
    >>> count_k(3, 3) # 3, 2 + 1, 1 + 2, 1 + 1 + 1
    4
    >>> count_k(4, 4)
    8
    >>> count_k(10, 3)
    274
    >>> count_k(300, 1) # Only one step at a time
    1
    """
    "*** YOUR CODE HERE ***"
```