FINAL REVIEW PART 1 (PRE-MT2) Solutions

COMPUTER SCIENCE MENTORS

April 19, 2021 - April 21, 2021

1 Environment Diagrams

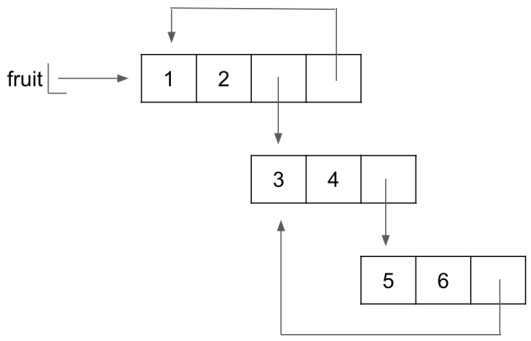
1. Draw the environment diagram that results from running the following code.

```
def f(f):
    def h(x, y):
        z = 4
        return lambda z:  (x + y) * z

    def g(y):
        nonlocal g, h
        g = lambda y: y[:4]
        h = lambda x, y: lambda f: f(x + y)
        return y[3] + y[5:8]

    return h(g("sarcasm!"), g("why?"))

f = f("61a")(2)
```

https://tinyurl.com/y56ezjz9

2. Fill in each blank in the code example below so that its environment diagram is the following. You do not need to use all the blanks.

```
fruit = [1, 2, [3, 4]]
fruit._____________________________
fruit[3][2]._____________________
fruit[2][2]._____________________
fruit[3][3][2][2][2][1] = ____
```




```
fruit = [1, 2, [3, 4]]
fruit.append(fruit)
fruit[3][2].append([5, 6])
fruit[2][2].append(fruit[2])
fruit[3][3][2][2][2][1] = 4
```

2 OOP

1. The `DLList` class is a spin off of the normal `Link` class we learned in class; each `DLList` link has a `prev` attribute that keeps track of the previous link and a **`next`** attribute that keeps track of the next link. Fill in the following methods for the `DLList` class.

   (a)
```
class DLList:
    """
    >>> lst = DLList(6, DLList(1))
    >>> lst.value
    6
    >>> lst.next.value
    1
    >>> lst.prev.value
    AttributeError: 'NoneType' object has no attribute 'value
       '
    """
    empty = None
    def __init__(self, value, next=empty, prev=empty):

        _____

        _____

        _____
```

```
    def __init__(self, value, next=empty, prev=empty):
        self.value = value
        self.next = next
        self.prev = prev
```

   (b)
```
    def add_last(self, value):
        """
        >>> lst = DLList(6)
        >>> lst.add_last(1)
        >>> lst.value
        6
        >>> lst.next.value
        1
        >>> lst.next.prev.value
        6
        """
```

```
        pointer = self
        while _____________________________________:

          ___________________________________________

        _________________ = DLList(_____________________________________)
```

```
    def add_last(self, value):
      pointer = self
      while pointer.next != DLList.empty:
        pointer = pointer.next
      pointer.next = DLList(value, DLList.empty, pointer)
```

(c)

```
    def add_first(self, value):
      """
      >>> lst = DLList('A')
      >>> lst.add_first(1)
      >>> lst.value
      1
      >>> lst.next.value
      'A'
      >>> lst.next.prev.value
      1
      >>> lst.add_first(6)
      >>> lst.value
      6
      >>> lst.next.next.prev.value
      1
      """
      old_first = DLList(_____________________________)

      _________________ = _____________________________________

      _________________ = _____________________________________

      if ___________________________________:

        ___________________________________________________________
```

```
    def add_first(self, value):
      old_first = DLList(self.value, self.next, self)
      self.value = value
      self.next = old_first
```

```
        if old_first.next != DLList.empty:
          old_first.next.prev = old_first
```

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3 Complexity

1. **Fast Exponentiation:** in this problem, we will examine a real-world algorithm used to improve the speed of calculating exponents.

   (a) First, express the runtime of the naive exponentiation algorithm in big-θ notation.

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n - 1)
```

   $\theta(n)$. n decreases by 1 each call, so there are naturally n calls.

   (b) Now, express the runtime of the fast exponentiation algorithm in big-θ notation.

```
def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0: # Assume square runs in constant time
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

   $\theta(\log n)$. n is halved each call, so the number of calls is the number of times n must be halved to get to 1. This is $\log n$.

   (c) What about this slightly modified version of `fast_exp`?

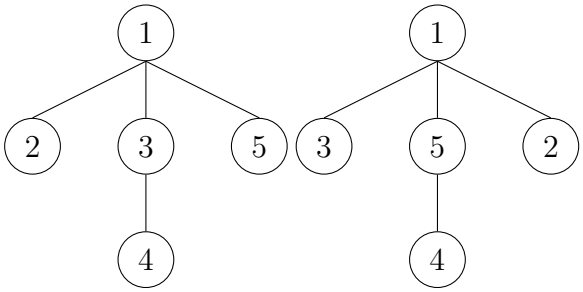
```
def fast_exp(b, n):
    for _ in range(50 * n):
        print("Killing time")
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n // 2))
    else:
        return b * fast_exp(b, n - 1)
```

   $\theta(n)$. Ignore the constant term. The first call will perform n operations, the second call will perform $n/2$ operations, the third will perform $n/4$ operations, etc. Using geometric series, we see this adds up to $2n$, which is n if we ignore constant terms.

4 Trees

1. Implement `rotate`, which takes in a tree and rotates the labels at each level of the tree by one to the left destructively. This rotation should be modular (That is, the leftmost label at a level will become the rightmost label after running rotate). You do NOT need to rotate across different branches.

   For example, given tree `t` on the left, `rotate(t)` should mutate `t` to give us the right.




```
def rotate(t):
    """
    >>> t1 = Tree(1, [Tree(2), Tree(3, [Tree(4)]), Tree(5)])
    >>> rotate(t1)
    >>> t1
    Tree(1, [Tree(3), Tree(5, [Tree(4)]), Tree(2)])
    >>> t2 = Tree(1, [Tree(2, [Tree(3), Tree(4)]),
                      Tree(5, [Tree(6)])])
    >>> rotate(t2)
    >>> t2
    Tree(1, [Tree(5, [Tree(4), Tree(3)]),
                      Tree(2, [Tree(6)])])
    """
    branch_labels = ____________________________________________

    n = len(t.branches)

    for ____________________________________________:

        ____________________________________________________

        ____________________________________________________

        ____________________________________________________
```

```
def rotate(t):
    branch_labels = [b.label for b in t.branches]
    n = len(t.branches)
    for i in range(n):
        branch = t.branches[i]
        branch.label = branch_labels[(i + 1) % n]
        rotate(branch)
```

5 Generators

1. (a) Implement `n_apply`, which takes in 3 inputs `f`, `n`, `x`, and outputs the result of applying `f`, a function, `n` times to `x`. For example, for `n = 3`, output the result of `f(f(f(x)))`.

```
def n_apply(f, n, x):
    """
    >>> n_apply(lambda x: x + 1, 3, 2)
    5
    """

    for _____________________________:

        x = _____________________________

    return _____________________________
```

```
def n_apply(f, n, x):
    for i in range(n):
        x = f(x)
    return x
```

   (b) Now implement `list_gen`, which takes in some list of integers `lst` and a `function f`. For the element at index `i` of `lst`, `list_gen` should apply `f` to the element `i` times and yield this value `lst[i]` times. You may use `n_apply` from the previous part.

```
def list_gen(lst, f):
    """
    >>> a = list_gen([1, 2, 3], lambda x: x + 1)
    >>> list(a)
    [1, 3, 3, 5, 5, 5]
    """
```

```
    for _____________________________:

        yield from [_____________________________________________]

def list_gen(lst, f):
    for i in range(len(lst)):
        yield from [n_apply(f, i, lst[i]) for j in range(lst[i
            ])]
```

2. Complete the implementation of `iter_link`, which takes in a linked list and returns a generator which will iterate over the values of the linked list in order. Your function should support deep linked lists.

```
def iter_link(lnk):
    """
    Yield the values of a linked list in order; your function
       should support deep linked lists.
    >>> lst1 = Link(1, Link(2, Link(3, Link(4))))
    >>> list(iter_link(lst1))
    [1, 2, 3, 4]
    >>> lst2 = Link(1, Link(Link(2, Link(3)), Link(4, Link(5))))
    >>> print(lst2)
    <1 <2 3> 4 5>
    >>> iter_lst2 = iter_link(lst2)
    >>> next(iter_lst2)
    1
    >>> next(iter_lst2)
    2
    >>> next(iter_lst2)
    3
    >>> next(iter_lst2)
    4
    """
    if lnk is not Link.empty:

        if type(_____________________) is Link:

            ____________________________________________

        else:

            ____________________________________________

        ____________________________________________
```

```
def iter_link(lnk):
    if lnk is not Link.empty:
        if type(lnk.first) is Link:
            yield from iter_link(lnk.first)
        else:
            yield lnk.first
        yield from iter_link(lnk.rest)
```

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