CS 61A
Fall 2021

# Linked Lists, Trees

Discussion 8: October 20, 2021 Solutions

## Representation: Repr, Str

**Q1: WWPD: Repr-esentation**

```
class A:
    def __init__(self, x):
        self.x = x

    def __repr__(self):
         return self.x

    def __str__(self):
         return self.x * 2

class B:
    def __init__(self):
         print('boo!')
         self.a = []

    def add_a(self, a):
         self.a.append(a)

    def __repr__(self):
         print(len(self.a))
         ret = ''
         for a in self.a:
             ret += str(a)
         return ret
```

Given the above class definitions, what will the following lines output?

```
>>> A('one')
```

one

```
>>> print(A('one'))
```

oneone

```
>>> repr(A('two'))
```

'two'

```
>>> b = B()
```

boo!

```
>>> b.add_a(A('a'))
>>> b.add_a(A('b'))
>>> b
```

2

aabb

# Linked Lists

There are many different implementations of sequences in Python. Today, we'll explore the linked list implementation.

A linked list is either an empty linked list, or a Link object containing a `first` value and the `rest` of the linked list.

To check if a linked list is an empty linked list, compare it against the class attribute `Link.empty`:

```
if link is Link.empty:
    print('This linked list is empty!')
else:
    print('This linked list is not empty!')
```

You can find an implementation of the `Link` class below:

```
class Link:
    """A linked list."""
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

### Q2: The Hy-rules of Linked Lists

In this question, we are given the following Linked List:

```
ganondorf = Link('zelda', Link('link', Link('sheik', Link.empty)))
```

What expression would give us the value `'sheik'` from this Linked List?

```
ganondorf.rest.rest.first
```

What is the value of `ganondorf.rest.first`?

```
'link'
```

What would be the value of `str(ganondorf)`?

```
'<zelda link sheik>'
```

What expression would mutate this linked list to `<zelda ganondorf sheik>`?

```
ganondorf.rest.first = 'ganondorf'
```

### Q3: Sum Nums

Write a function that takes in a linked list and returns the sum of all its elements. You may assume all elements in `s` are integers. Try to implement this recursively!

```
def sum_nums(s):
    """
    >>> a = Link(1, Link(6, Link(7)))
    >>> sum_nums(a)
    14
    """
    if s == Link.empty:
        return 0
    return s.first + sum_nums(s.rest)
```

### Q4: Multiply Links

Write a function that takes in a Python list of linked lists and multiplies them element-wise. It should return a new linked list.

If not all of the `Link` objects are of equal length, return a linked list whose length is that of the shortest linked list given. You may assume the `Link` objects are shallow linked lists, and that `lst_of_lnks` contains at least one linked list.

```
def multiply_lnks(lst_of_lnks):
    """
    >>> a = Link(2, Link(3, Link(5)))
    >>> b = Link(6, Link(4, Link(2)))
    >>> c = Link(4, Link(1, Link(0, Link(2))))
    >>> p = multiply_lnks([a, b, c])
    >>> p.first
    48
    >>> p.rest.first
    12
    >>> p.rest.rest.rest is Link.empty
    True
    """
    # Implementation Note: you might not need all lines in this
    skeleton code
    product = 1
    for lnk in lst_of_lnks:
        if lnk is Link.empty:
            return Link.empty
        product *= lnk.first
    lst_of_lnks_rests = [lnk.rest for lnk in lst_of_lnks]
    return Link(product, multiply_lnks(lst_of_lnks_rests))
```

For our base case, if we detect that any of the lists in the list of `Link`s is empty, we can return the empty linked list as we're not going to multiply anything.

Otherwise, we compute the product of all the firsts in our list of `Link`s. Then, the subproblem we use here is the rest of all the linked lists in our list of Links. Remember that the result of calling `multiply_lnks` will be a linked list! We'll use the product we've built so far as the first item in the returned `Link`, and then the result of the recursive call as the rest of that `Link`.

Next, we have the iterative solution:

```
def multiply_lnks(lst_of_lnks):
    """
    >>> a = Link(2, Link(3, Link(5)))
    >>> b = Link(6, Link(4, Link(2)))
    >>> c = Link(4, Link(1, Link(0, Link(2))))
    >>> p = multiply_lnks([a, b, c])
    >>> p.first
    48
    >>> p.rest.first
    12
    >>> p.rest.rest.rest is Link.empty
    True
    """
    # Alternate iterative approach
    import operator
    from functools import reduce
    def prod(factors):
         return reduce(operator.mul, factors, 1)

    head = Link.empty
    tail = head
    while Link.empty not in lst_of_lnks:
        all_prod = prod([l.first for l in lst_of_lnks])
        if head is Link.empty:
            head = Link(all_prod)
            tail = head
        else:
            tail.rest = Link(all_prod)
            tail = tail.rest
        lst_of_lnks = [l.rest for l in lst_of_lnks]
    return head
```

The iterative solution is a bit more involved than the recursive solution. Instead of building the list **backwards** as in the recursive solution (because of the order that the recursive calls result in, the last item in our list will be finished first), we'll build the resulting linked list as we go along.

We use`head` and `tail` to track the front and end of the new linked list we're creating. Our stopping condition for the loop is if any of the `Link`s in our list of `Link`s runs out of items.

Finally, there's some special handling for the first item. We need to update both head and tail in that case. Otherwise, we just append to the end of our list using tail, and update tail.

### Q5: Flip Two

Write a recursive function `flip_two` that takes as input a linked list `s` and mutates `s` so that every pair is flipped.

```
def flip_two(s):
    """
    >>> one_lnk = Link(1)
    >>> flip_two(one_lnk)
    >>> one_lnk
    Link(1)
    >>> lnk = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> flip_two(lnk)
    >>> lnk
    Link(2, Link(1, Link(4, Link(3, Link(5)))))
    """
    # Recursive solution:
    if s is Link.empty or s.rest is Link.empty:
        return
    s.first, s.rest.first = s.rest.first, s.first
    flip_two(s.rest.rest)


    # For an extra challenge, try writing out an iterative approach
    as well below!
    return # separating recursive and iterative implementations

    # Iterative approach
    while s is not Link.empty and s.rest is not Link.empty:
        s.first, s.rest.first = s.rest.first, s.first
        s = s.rest.rest
```

If there's only a single item (or no item) to flip, then we're done.

Otherwise, we swap the contents of the first and second items in the list. Since we've handled the first two items, we then need to recurse on

Although the question explicitly asks for a recursive solution, there is also a fairly similar iterative solution (see python solution).

We will advance `s` until we see there are no more items or there is only one more Link object to process. Processing each `Link` involves swapping the contents of the first and second items in the list (same as the recursive solution).

# Trees

We define a tree to be a recursive data abstraction that has a label (the value stored in the root of the tree) and branches (a list of trees directly underneath the root). Previously, we implemented the tree abstraction using Python lists. Let's look at another implementation using objects instead:

```
class Tree:
    def __init__(self, label, branches=[]):
        for b in branches:
            assert isinstance(b, Tree)
        self.label = label
        self.branches = branches

    def is_leaf(self):
        return not self.branches
```

With this implementation, we can mutate a tree using attribute assignment, which wasn't possible in the previous implementation using lists. That's why we sometimes call these objects "mutable trees."

```
>>> t = Tree(3, [Tree(4), Tree(5)])
>>> t.label = 5
>>> t.label
5
```

### Q6: Make Even

Define a function `make_even` which takes in a tree `t` whose values are integers, and mutates the tree such that all the odd integers are increased by 1 and all the even integers remain the same.

```
def make_even(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(3)]), Tree(4), Tree(5)])
    >>> make_even(t)
    >>> t.label
    2
    >>> t.branches[0].branches[0].label
    4
    """
    if t.label % 2 != 0:
        t.label += 1
    for branch in t.branches:
        make_even(branch)
    return
    # Alternate Solution

    t.label += t.label % 2
    for branch in t.branches:
        make_even(branch)
    return
```

### Q7: Leaves

Write a function `leaves` that returns a list of all the label values of the leaf nodes of a `Tree`.

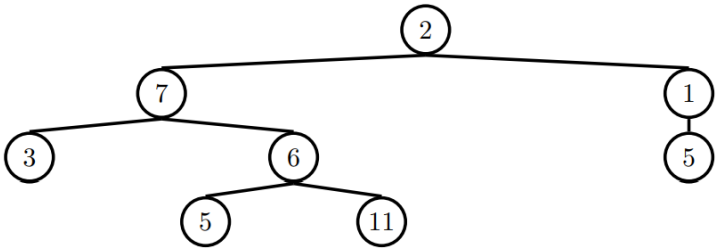
```
def leaves(t):
    """Returns a list of all the labels of the leaf nodes of the
    Tree t.

    >>> leaves(Tree(1))
    [1]
    >>> leaves(Tree(1, [Tree(2, [Tree(3)]), Tree(4)]))
    [3, 4]
    """
    if t.is_leaf():
        return [t.label]
    all_leaves = []
    for b in t.branches:
        all_leaves += leaves(b)
    return all_leaves
```

### Q8: Find Paths

**Hint**: This question is similar to `find_path` on Discussion 05.

Define the procedure `find_paths` that, given a Tree `t` and an `entry`, returns a list of lists containing the nodes along each path from the root of `t` to `entry`. You may return the paths in any order.

For instance, for the following tree `tree_ex`, `find_paths` should behave as specified




in the function doctests.

```
def find_paths(t, entry):
    """
    >>> tree_ex = Tree(2, [Tree(7, [Tree(3), Tree(6, [Tree(5), Tree
    (11)])]), Tree(1, [Tree(5)])])
    >>> find_paths(tree_ex, 5)
    [[2, 7, 6, 5], [2, 1, 5]]
    >>> find_paths(tree_ex, 12)
    []
    """

    paths = []
    if t.label == entry:
        paths.append([t.label])
    for b in t.branches:
        for path in find_paths(b, entry):
            paths.append([t.label] + path)
    return paths
```