CS61C

Great Ideas in Computer Architecture

(a.k.a. Machine Structures)

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Number Representation

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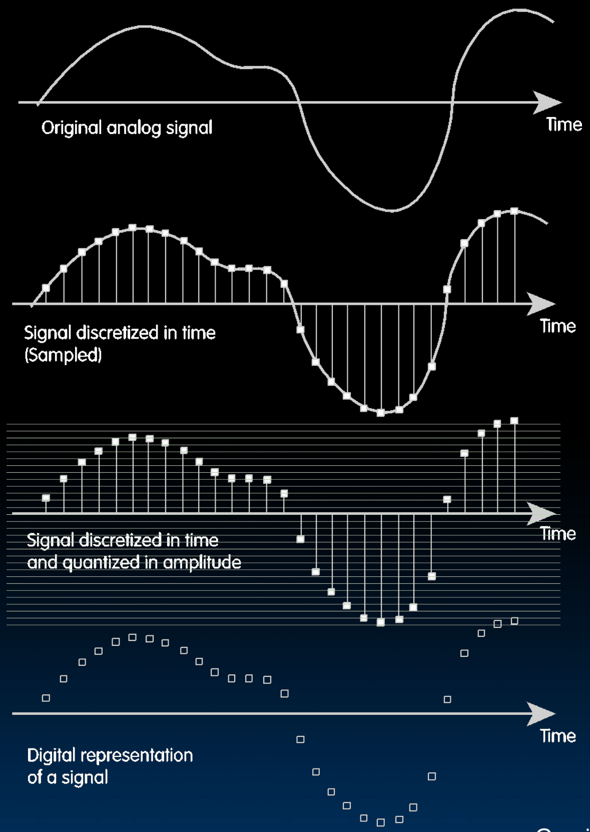
Data input: Analog → Digital

- Real world is analog!
- To import analog information, we must do two things
  - Sample
    - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
  - Quantize
    - For every one of these samples, we figure out where, on a 16-bit (65,536 tic-mark) "yardstick", it lies.



Digital data not necessarily born Analog…

hof.povray.org

BIG IDEA: Bits can represent anything!!

- Characters?
  - 26 letters ⇒ 5 bits ($2^5 = 32$)
  - upper/lower case + punctuation ⇒ 7 bits (in 8) ("ASCII")
  - standard code to cover all the world's languages ⇒ 8,16,32 bits ("Unicode") `www.unicode.com`
- Logical values?
  - 0 → False, 1 → True
- colors ? Ex: *Red (00)* *Green (01)* *Blue (11)*
- locations / addresses? commands?
- MEMORIZE: N bits ⇔ at most 2^N things

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Binary
Decimal
Hex

Base 10 (Ten) #s, Decimals

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

$3271 = 3271_{10} =$

$(3 \text{x} 10^3) + (2 \text{x} 10^2) + (7 \text{x} 10^1) + (1 \text{x} 10^0)$

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Base 2 (Two) #s, Binary (to Decimal)

Digits: 0, 1 (**bi**nary digi**ts** □ bits)

Example: "1101" in binary? ("0b1101")

$$1101_2 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$= 8 + 4 + 0 + 1$$

$$= 13$$

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Base 16 (Sixteen) #s, Hexadecimal (to Decimal)

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
10,11,12,13,14,15

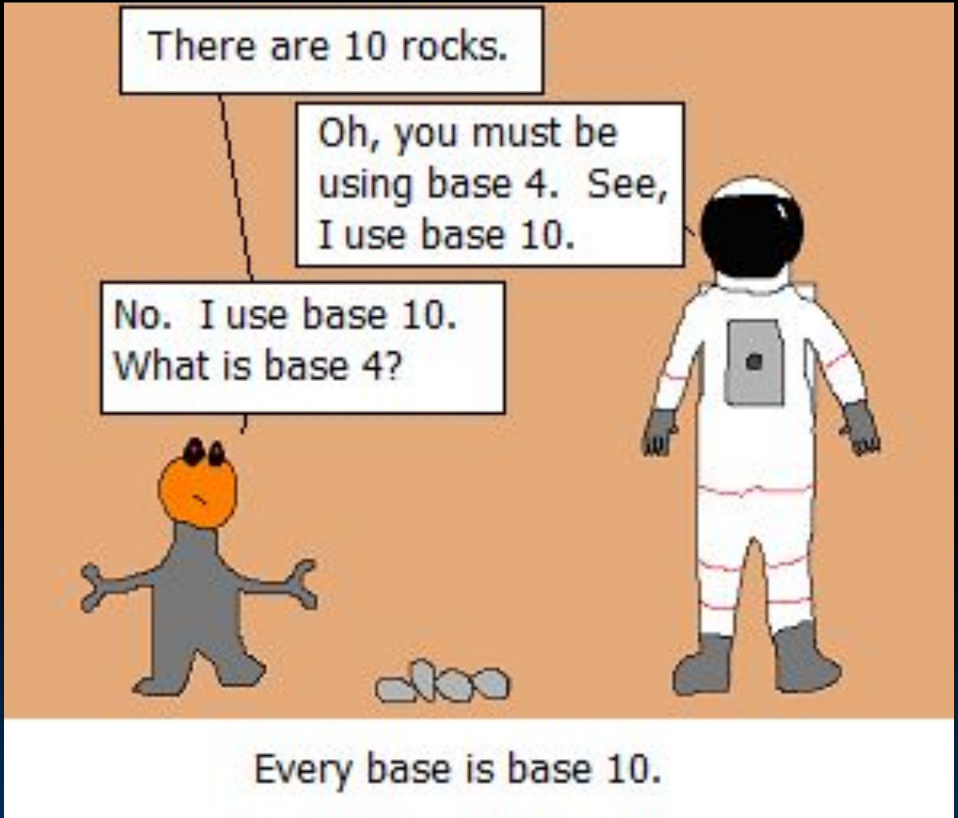
Example: "A5" in Hexadecimal?

$0xA5 = A5_{16} = (10 \times 16^1) + (5 \times 16^0)$

$= 160 + 5$

$= 165$

Every Base is Base 10…



Convert from Decimal to Binary

- E.g., 13 to binary?
- Start with the columns

~~1~~
~~5~~
~~1~~
0

2^3=8	2^2=4	2^1=2	2^0=1
1	1	0	1

- Left to right, is (column) ≤ number **n**?
  - If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
  - If not, put 0 and keep going. (and Stop at 0)

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Convert from Decimal to Hexadecimal

- E.g., 165 to hexadecimal?
- Start with the columns

~~16~~
~~5~~
0

16^3=4096	16^2=256	16^1=16	16^0=1
0	0	(10) A	5

- Left to right, is (column) ≤ number **n**?
  - If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
  - If not, put 0 and keep going. (and Stop at 0)

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Convert Binary Hexadecimal

- Binary Hex? Easy!
  - Always left-pad with 0s to make full 4-bit values, then look up!
  - E.g., `0b11110` to Hex?
    - `0b11110` `0b00011110`
    - Then look up: `0x1E`
- Hex Binary? Easy!
  - Just look up, drop leading 0s
    - `0x1E0b000111100b11110`

D	H	B
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Decimal vs Hexadecimal vs Binary

- 4 Bits
  - 1 "Nibble"
  - 1 Hex Digit = 16 things
- 8 Bits
  - 1 "Byte"
  - 2 Hex Digits = 256 things
  - Color is usually
    0-255 Red,
    0-255 Green,
    0-255 Blue.
    #D0367F=

D	H	B
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Which base do we use?

- Decimal: great for humans, especially when doing arithmetic
- Hex: if human looking at long strings of binary numbers, its much easier to convert to hex and see 4 bits/symbol
  - Terrible for arithmetic on paper
- Binary: what computers use;
  you will learn how computers do +, -, *, /
  - To a computer, numbers always binary
  - Regardless of how number is written:
  - $32_{ten} == 32_{10} ==$ 0x20 $== 100000_2 ==$ 0b100000
  - Use subscripts "ten", "hex", "two" in book, slides when might be confusing

The computer knows it, too…

```c
#include <stdio.h>

int main() {
    const int N = 1234;

    printf("Decimal: %d\n", N);
    printf("Hex:     %x\n", N);
    printf("Octal:   %o\n", N);

    printf("Literals (not supported by all compilers):\n");
    printf("0x4d2         = %d (hex)\n", 0x4d2);
    printf("0b10011010010 = %d (binary)\n", 0b10011010010);
    printf("02322         = %d (octal, prefix 0 - zero)\n", 02322);
}
```

Output

```
Decimal: 1234
Hex:     4d2
Octal:   2322
Literals (not supported by all compilers):
0x4d2         = 1234 (hex)
0b10011010010 = 1234 (binary)
02322         = 1234 (octal, prefix 0 - zero)
```


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Number Representations

What to do with representations of numbers?

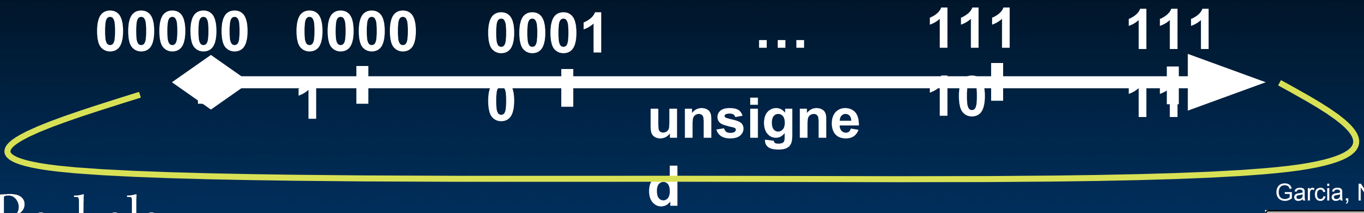
- What to do with number representations?
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them

$$\begin{array}{ccccc} & 1 & 0 & 1 & 0 \\ + & 0 & 1 & 1 & 1 \\ \hline \end{array}$$

- Example: 10 + 7 = 17
  - …so simple to add in binary that we can build circuits to do it!
  - Subtraction just as you would in decimal
  - Comparison: How do you tell if X > Y ?

What if too big?

- Binary bit patterns are simply representatives of numbers. Abstraction!
  - Strictly speaking they are called "numerals".
- Numerals really have an ∞ number of digits
  - with almost all being same (00…0 or 11…1) except for a few of the rightmost digits
  - Just don't normally show leading digits
- If result of add (or -, *, /) cannot be represented by these rightmost HW bits, we say overflow occurred



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How to Represent Negative Numbers?

(C's `unsigned int`, C18's `uintN_t`)

- So far, unsigned numbers

Binary odometer

00000   00001   ...   01111   10000   ...   11111

- Obvious solution: define leftmost bit to be sign!
  - □ 0 □ +   1 □ –   ...and rest of bits are numerical value
- Representation called Sign and Magnitude

Binary odometer

00000   00001   ...   01111

11111   ... 10001   10000

META: Ain't no free lunch

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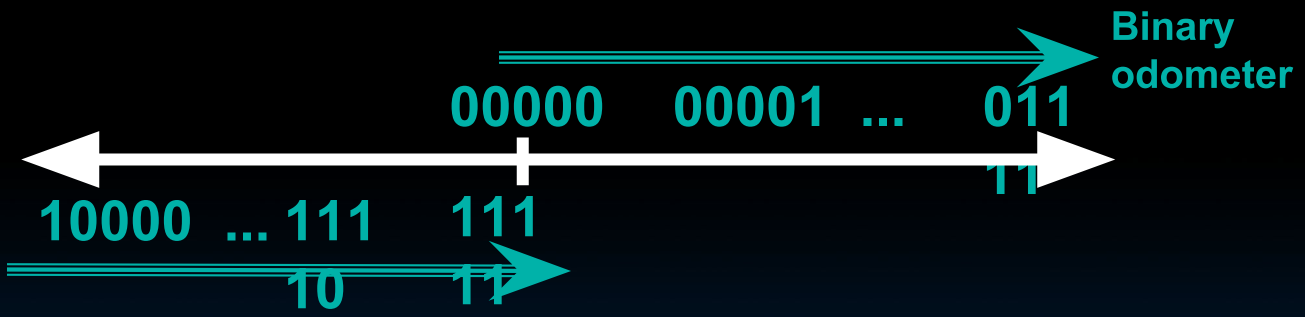
Shortcomings of Sign and Magnitude?

- Arithmetic circuit complicated
  - Special steps depending on if signs are the same or not
- Also, two zeros
  - 0x00000000 = $+0_{ten}$
  - 0x80000000 = $–0_{ten}$
  - What would two 0s mean for programming?
- Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases!
- Therefore sign and magnitude used only in signal processors

Another try: complement the bits

- Example: $7_{10} = 00111_2$ $-7_{10} = 11000_2$
- Called One's Complement
- Note: positive numbers have leading 0s, negative numbers have leadings 1s.



- What is -00000 ? Answer: 11111
- How many positive numbers in N bits?
- How many negative numbers?

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Shortcomings of One's Complement?

- Arithmetic still somewhat complicated
- Still two zeros
  - $0x00000000 = +0_{ten}$
  - $0xFFFFFFFF = -0_{ten}$
- Although used for a while on some computer products, one's complement was eventually abandoned because another solution was better.

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Two's Complement & Bias Encoding

Standard Negative # Representation

- Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.
  - Solution! For negative numbers, complement, then add 1 to the result
- As with sign and magnitude, & one's compl. leading 0s → positive, leading 1s → negative
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1…1111 is -1, not -0 (as in sign & mag.)
- This representation is Two's Complement
  - This makes the hardware simple!

(C's `int`, C18's `intN_t`, aka a "signed integer")

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Two's Complement Formula

- Can represent positive and negative numbers in terms of the bit value times a power of 2:

  $d_{31} \times -(2^{31}) + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$

- Example: 1101_{two} in a nibble?

  $= 1 \times -(2^3) + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

  $= -2^3 + 2^2 + 0 + 2^0$

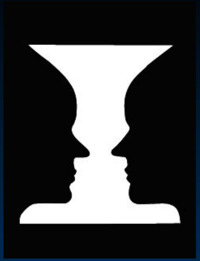
  $= -8 + 4 + 0 + 1$

  $= -8 + 5$

  $= -3_{ten}$

Example: -3 to +3 to -3 (again, in a nibble):

x : 1101_{two}
x' : 0010_{two}
+1 : 0011_{two}
()' : 1100_{two}
+1 : 1101_{two}



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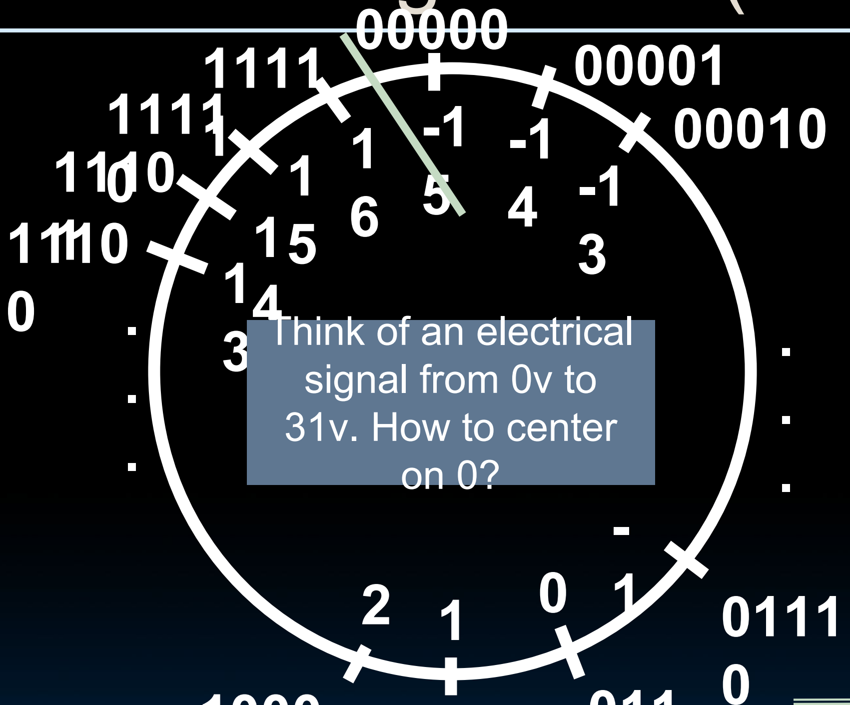
Two's Complement Number "line": N = 5

- 2^{N-1} non-negatives
- 2^{N-1} negatives
- one zero
- how many positives?

Bias Encoding: N = 5 (bias = -15)



- # = unsigned + bias
- Bias for N bits chosen as $-(2^{N-1}-1)$
- one zero
- how many positives?

And in summary...

META: We often make design decisions to make HW simple

- We represent "things" in computers as particular bit patterns: N bits ⇒ 2^N things
- These 5 integer encodings have different benefits; 1s complement and sign/mag have most problems.
- unsigned (C18's **`uintN_t`**) :

00000 00001 ... 01111 10000 ... 11111

- Two's complement (C99's **`intN_t`**) universal, learn!

00000 00001 ... 01111

10000 ... 11110 11111

- Overflow: numbe rrors!

META: Ain't no free lunch